

## Chapter 6

### Distributed and Microstrip Impedance Matching

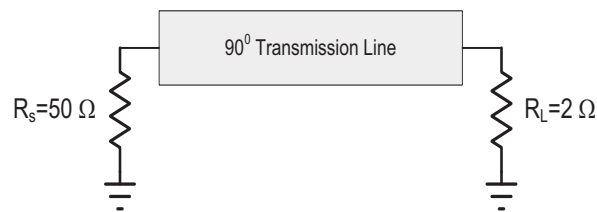
#### 6.1 Introduction

Distributed networks are comprised of transmission line elements rather than discrete resistors, inductors, and capacitors. These transmission lines can take the form of the various transmission lines covered in chapter 2. At RF and microwave frequencies, where the wavelengths of the signals become comparable to the physical dimensions of the components, even lumped elements behave like distributed components. At microwave frequencies the distributed networks are more realizable than the lumped element networks.

#### 6.2.2 Analytical Design of Quarter-Wave Matching Networks

In this section, based on the equations developed in Appendix C-1 to C-3, two quarter-wave matching networks for  $R_L = 2 \Omega$  and  $R_L = 150 \Omega$  are designed. For each case the loaded Q factor and bandwidth at 3 and 20 dB return loss are calculated.

**Example 6-1:** Design a quarter-wave network intended to match a  $50 \Omega$  source to a  $2 \Omega$  load at 100 MHz. Calculate the Q factor and the fractional bandwidths at 3 and 20 dB return loss. Compare the calculations with the simulation results in ADS.



**Figure 6-1** Matching quarter-wave transformer ( $R_L < R_s$ )

**Solution:** Using the equations developed in the Appendix C-1, the characteristic impedance, loaded Q factor, and the bandwidths of the matching network are calculated. The procedure follows:

1. Enter design parameters and normalize the load impedance

$$R_S=50; R_L=2; f=100\text{e}6, r=R_L/R_S$$

2. Use Equations (6-4) to (6-12) in Appendix C-1 to calculate the parameters

$$Z_1=R_S*\text{sqrt}(r)$$

$$\text{FBW}_{20\text{dB}}=2-(4/\text{PI})*\text{acos}((0.2*\text{sqrt}(r))/(\text{sqrt}(0.99)*(abs(1-r))))$$

$$\text{BW}_{20\text{dB}}=(f*\text{FBW}_{20\text{dB}})$$

$$\text{FBW}_{3\text{dB}}=2-(4/\text{PI})*\text{acos}((2*\text{sqrt}(r))/(abs(1-r))))$$

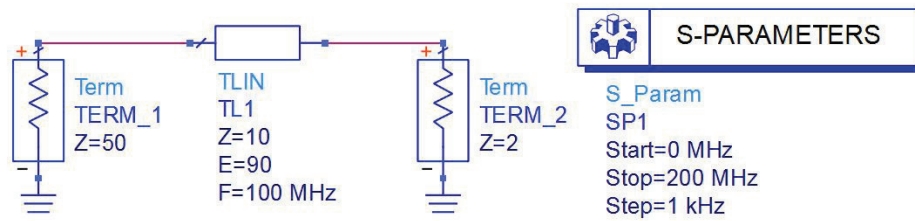
$$\text{BW}_{3\text{dB}}=(f*\text{FBW}_{3\text{dB}})$$

$$Q=1/\text{FBW}_{3\text{dB}}$$

The calculation results show that the characteristic impedance = 10  $\Omega$ , Q factor is 1.827, and the bandwidths at 3 dB and 20dB return loss are 54.72 MHz and 5.333 MHz, respectively.

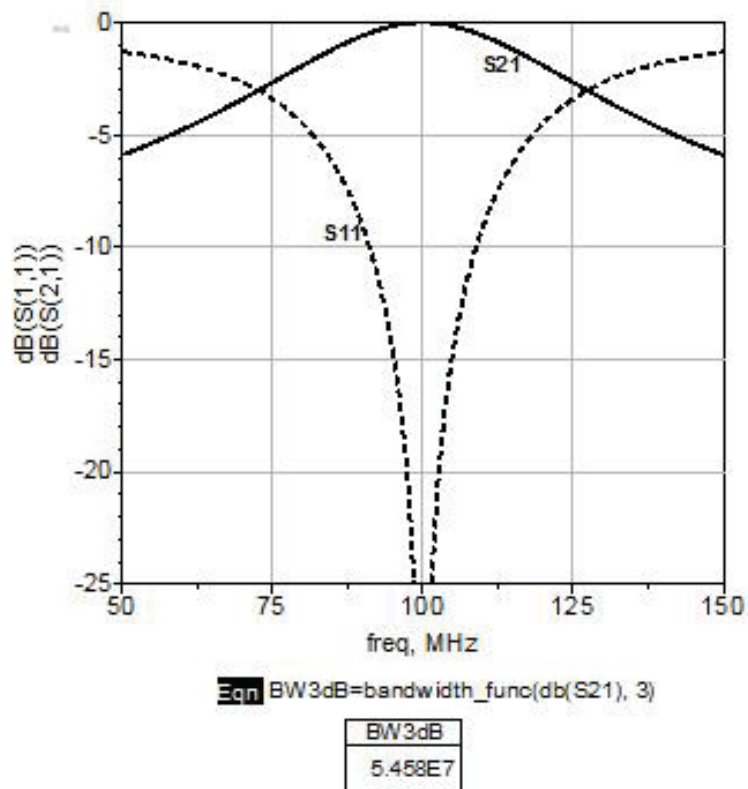
The fractional bandwidth at 20 dB return loss indicates that the matching network has a narrow bandwidth. This is characteristic of a matching network where the load resistor is much smaller than the source resistor.

To measure the same parameters in ADS, create a new workspace and open a new schematic window. Insert the S\_Params Template and place the TLIN component on the schematic. Set the component values as shown in Figure 6-2.



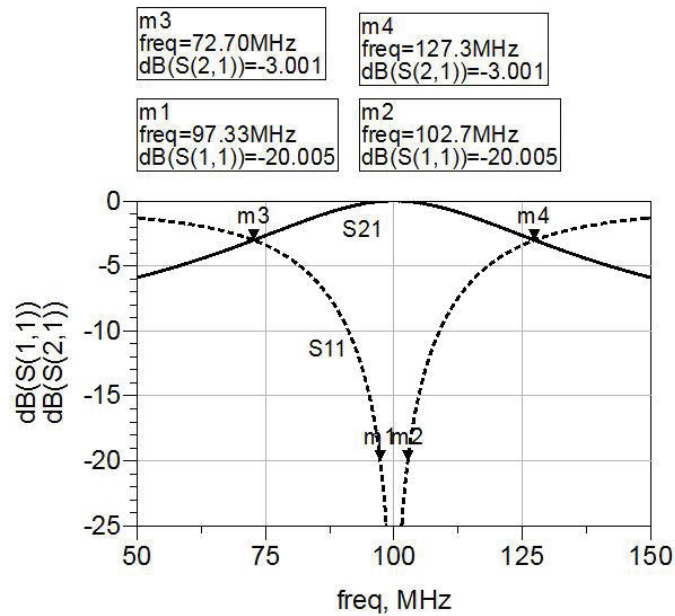
**Figure 6-2** Schematic of the quarter-wave matching network

Simulate the schematic and display the input reflection coefficient,  $S_{11}$ , and the forward transmission,  $S_{21}$ , in a rectangular plot. To measure the 3 dB bandwidth automatically in ADS, use the built-in equation for BW3dB, as shown in Figure 6-3.



**Figure 6-3** Network Response and 3 dB bandwidth

Figure 6-3 shows that the 3 dB bandwidth is 54.58 MHz. To measure the bandwidths manually, place markers at 3 and 20 dB return loss as shown in Figure 6-4.



**Figure 6-4** Bandwidth measurement at 3 and 20 dB return loss

Reading marker frequencies at 3 and 20 dB return loss, the corresponding bandwidths are:

$$BW_{-3dB} = 127.3 - 72.7 = 54.6 \quad \text{MHz}$$

$$BW_{-20dB} = 102.7 - 97.3 = 5.4 \quad \text{MHz}$$

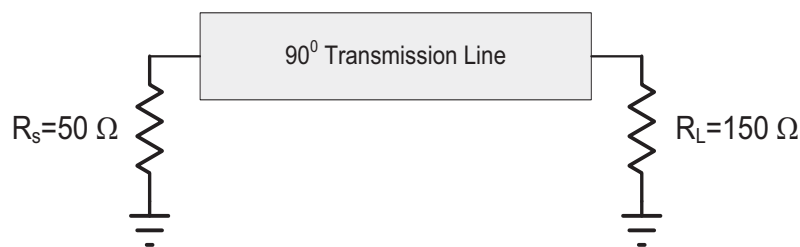
The Q factor of the matching network is measured by using the equation.

$$Q = \frac{1}{FBW_{-3dB}} \frac{\sqrt{(127.3)(72.7)}}{127.3 - 72.7} = 1.76$$

Note that the measured Q factor and bandwidths are in close agreement with the calculated values.

**Example 6-2:** Design a quarter-wave network intended to match a  $50\ \Omega$  source to a  $150\ \Omega$  load. The design frequency is 100 MHz. Compare the calculated Q factor and the fractional bandwidths, at 3 and 20 dB return loss, with the measurements.

**Solution:** The schematic of the quarter-wave matching network with  $R_s = 50$  and  $R_L = 150\ \Omega$  is shown in Figure 6-5.

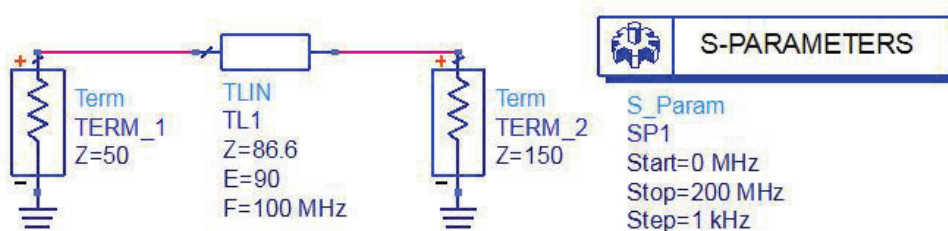


**Figure 6-5** Quarter-wave matching network ( $R_L > R_s$ )

The characteristic impedance of the quarter-wave matching network is:

$$Z_1 = \sqrt{(50)(150)} = 86.6\ \Omega$$

The schematic of the matching network in ADS is shown in Figure 6-6.



**Figure 6-6** Schematic of the matching quarter-wave network

Use equations developed in the Appendix C to calculate the characteristic impedance, the Q factor, and the bandwidths of the matching network at 3 and 20 dB return loss. The procedure follows.

1. Enter design parameters and normalize the load impedance

$$R_L=150; f=100\text{e}6, r=R_L/R_S$$

2. Use Equations (6-4) to (6-12) in Appendix C to calculate the parameters

$$Z_1=R_S*\text{sqrt}(r)$$

$$\text{FBW}_{20\text{dB}}=2-(4/\text{PI})*\text{acos}((0.2*\text{sqrt}(r))/(\text{sqrt}(0.99)*(abs(1-r))))$$

$$\text{BW}_{20\text{dB}}=(f*\text{FBW}_{20\text{dB}})$$

$$\text{FBW}_{3\text{dB}}=2-(4/\text{PI})*\text{acos}((2*\text{sqrt}(r))/(abs(1-r)))$$

$$\text{BW}_{3\text{dB}}=(f*\text{FBW}_{3\text{dB}})$$

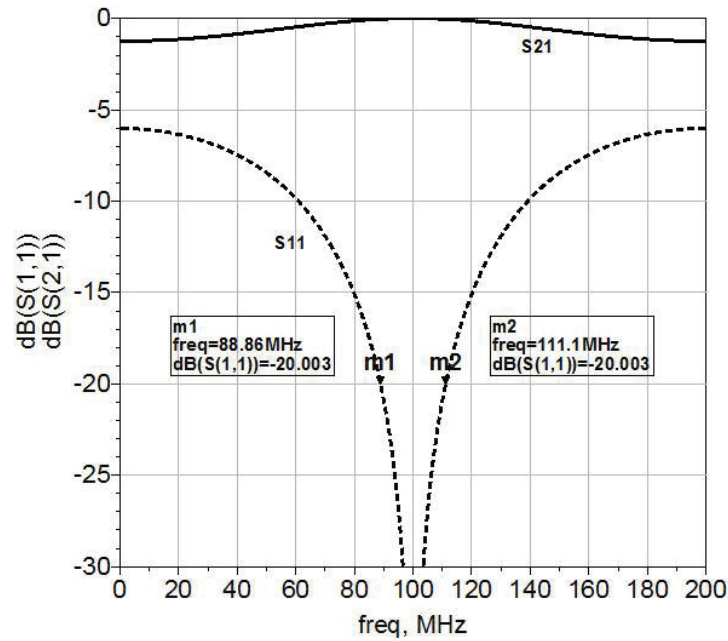
$$Q_e=1/\text{FBW}_{3\text{dB}}$$

Next simulate the schematic and display the input reflection coefficient, S11, and the forward transmission, S21, in a rectangular plot.

Place markers m1 and m2 at 20 dB return loss to measure the matching bandwidth at 20 dB return loss.

The calculation results in Figure 6-7 show that the bandwidth at 20 dB return loss is 22.24 MHz. This is over four times the bandwidth that was achieved with the 2 Ohm load impedance.

Also notice that the overall Q of the network has significantly increased due to higher load impedance.



**Figure 6-7** Response of the quarter-wave matching network

Figure 6-7 shows the measured bandwidth at 20 dB return loss is:

$$111.1 - 88.86 = 22.24 \text{ MHz}$$

## Quarter-Wave Matching Bandwidth

We have shown that the achievable bandwidth in a quarter-wave matching network is related to the ratio of the load to the source impedance (mismatch ratio) as well as the value of the input reflection coefficient. It is insightful to examine the relationship between these quantities when one of the parameters is swept in value.

## Quarter-Wave Matching Bandwidth and Power Loss

The fractional bandwidth and power loss of a quarter-wave matching network can be calculated as a function of the input reflection coefficient,  $\Gamma_{IN}$ , and the normalized load resistor. The procedure follows.

1. Enter the equations for the input reflection coefficient, the power loss, and the conversion of reflection coefficient to return loss in dB.
2. Enter the desired values for the source and load resistors
3. Normalize the load resistor with respect to source resistor
4. Use Equation (6-6) in Appendix C to calculate the fractional bandwidth.

**Example 6 - 3:** For a 50 Ohm source and 2 Ohm load resistors, calculate the fractional bandwidth and power loss from  $\Gamma = 0.1$  to  $\Gamma = 0.707$ .

**Solution:** Follow the above procedure to calculate the fractional bandwidth and power loss when  $R_S = 50 \Omega$  and  $R_L = 2 \Omega$ .

1. Write the reflection coefficient, power loss, and return loss equations

$$V_{\text{swr}} = \text{Sweep1\_Data.VSWR1}$$

$$\text{ReflCoef} = (v_{\text{swr}} - 1) / (v_{\text{swr}} + 1)$$

$$\text{Powerloss} = (1 - (1 - (\text{abs}(\text{ReflCoef})^2))) * 100$$

$$\text{RLdB} = -20 * \log(\text{abs}(\text{ReflCoef}))$$

1. Enter the source and load resistor values

$$R_S = 50; R_L = 2, r = R_L / R_S$$

3. Calculate the Fractional Bandwidth

$$\text{FBW} = (2 - (4/\pi) * \arccos((2 * (\text{ReflCoef}) * \sqrt{r}) / (\sqrt{(1 - (\text{ReflCoef})^2)} * \text{abs}(1 - r))))$$

Sweep the parameters to display the fractional bandwidth and power loss for reflection coefficients from 0.1 to 0.707, as shown in Table 6-1.



	ReflCoef	FBW ...	RLdB	Powerloss
1	0.1	0.053	20	1
2	0.11	0.059	19.172	1.21
3	0.12	0.064	18.417	1.44
4	0.13	0.07	17.721	1.69
5	0.14	0.075	17.077	1.96
6	0.15	0.081	16.478	2.25
7	0.16	0.086	15.917	2.56
8	0.17	0.092	15.391	2.89
9	0.18	0.097	14.895	3.24
10	0.19	0.103	14.425	3.61
11	0.2	0.108	13.979	4
12	0.3	0.167	10.458	9
13	0.4	0.233	7.959	16
14	0.5	0.309	6.021	25
15	0.6	0.405	4.437	36
16	0.707	0.547	3.012	49.985

**Table 6-1** Fractional bandwidth and power loss for  $R_L=2\ \Omega$

Table 6-1 shows that the fractional bandwidths at 0.1 reflection coefficient, corresponding to 20 dB return loss, is 5.3 % with 1% power loss while at 0.707 reflection coefficient, corresponding to 3 dB return loss, the fractional bandwidth is 54.7 % with 49.985 % power loss. Notice that the fractional bandwidths, at 3 and 20 dB return loss, are the same as calculated in Example 6 -1.

**Example 6-4:** For a  $50\ \Omega$  source and  $150\ \Omega$  load resistors, calculate the fractional bandwidth and power loss from  $\Gamma = 0.1$  to  $\Gamma = 0.707$ .

**Solution:** Following the procedure, the calculations are shown in Figure 6-15. Sweep the parameters and display the result, as shown in Table 6-3.

1. Enter reflection coefficient, powerloss, and return loss Equations

$$vswr = \text{Sweep1\_Data.VSWR1}$$

$$\text{ReflCoef} = (vswr - 1) / (vswr + 1)$$

$$\text{Powerloss} = (1 - (1 - (\text{abs}(\text{ReflCoef})^2))) * 100 \quad \text{RLdB} = -20 * \log(\text{abs}(\text{ReflCoef}))$$

2. Enter the source and load resistor values and normalized load

$$R_S=50; R_L=150; r=R_L/R_S$$

2. Calculate the Fractional Bandwidth

$$FBW=(2-(4/\pi)*\cos((2*(\text{ReflCoef})*\sqrt{r})/(\sqrt{(1-(\text{ReflCoef})^2))*\text{abs}(1-r))))$$

The values of the reflection coefficient, power loss, and return loss are given in the Table.6-2.

.	ReflCoef	FBW (rad)	RLdB	Powerloss
1	0.1	0.223	20	1
2	0.11	0.246	19.172	1.21
3	0.12	0.269	18.417	1.44
4	0.13	0.292	17.721	1.69
5	0.14	0.315	17.077	1.96
6	0.15	0.339	16.478	2.25
7	0.16	0.362	15.917	2.56
8	0.17	0.386	15.391	2.89
9	0.18	0.411	14.895	3.24
10	0.19	0.435	14.425	3.61
11	0.2	0.46	13.979	4
12	0.3	0.733	10.458	9
13	0.4	1.091	7.959	16
14	0.5	2	6.021	25
15	0.6	1.#QO	4.437	36
16	0.707	1.#QO	3.012	49.985

**Table 6-2** Fractional bandwidth and power loss for  $R_S=50 \Omega$  and  $R_L=150 \Omega$

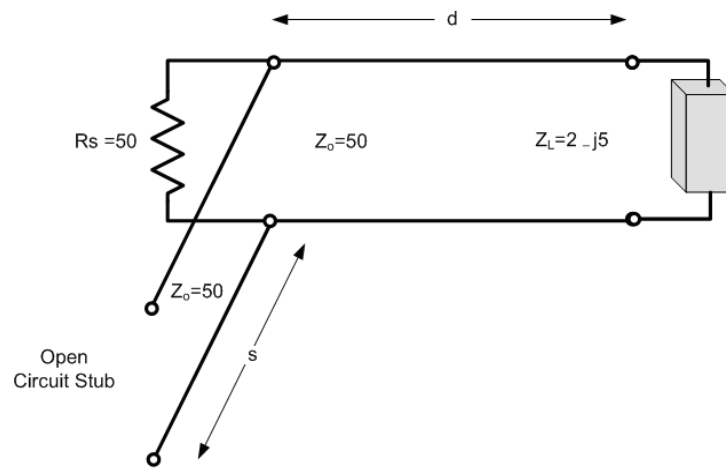
Table 6-2 shows that the fractional bandwidth at  $\Gamma = 0.1$ , corresponding to 20 dB return loss, is 22.3 % with a 1% power loss.

Notice that the fractional bandwidth is the same as calculated in the previous example. The higher fractional bandwidth in this example, compared to its value in the previous example is due to the lower ratio of the load to source resistance.

## Single-Stub Matching Network Design

**Example 6-5:** Design a open-circuited single-stub network to match a load resistance  $Z_L = 2 - j5 \Omega$  to a resistive source  $R_S = 50 \Omega$  at 100 MHz. Calculate the fractional bandwidths of the matching network at 3 and 20 dB return loss. Display the response and compare the calculations with measurements.

**Solution:** The design example is shown in Figure 6-8.



**Figure 6-8** Complex load to resistive source matching network

Using Equations in Appendix C to calculate the electrical lengths of the line and stub matching transmission lines. The calculation in Matlab script follows .

1. Enter design parameters and normalized load impedance

$$R_S=50; R_L=2; X_L=-5; f=100e6; r=R_L/R_S; x=X_L/R_S$$

2. Use Equations (6-18) to (6-29) in Appendix C to calculate  $t_1$ ,  $t_2$ ,  $d_1$  and  $d_2$ ,  $B_1$ ,  $B_2$ ,  $so_1$  and  $so_2$ .

$$t_1=(x+\sqrt{r*(r^2+x^2-2*r+1)}))/(r-1)$$

$$t2=(x-\sqrt{r*(r^2+x^2-2*r+1)})/(r-1)$$

$$d1=360*(\text{atan}(t1))/(2*\pi)$$

$$d2=360*(\text{atan}(t2))/(2*\pi)$$

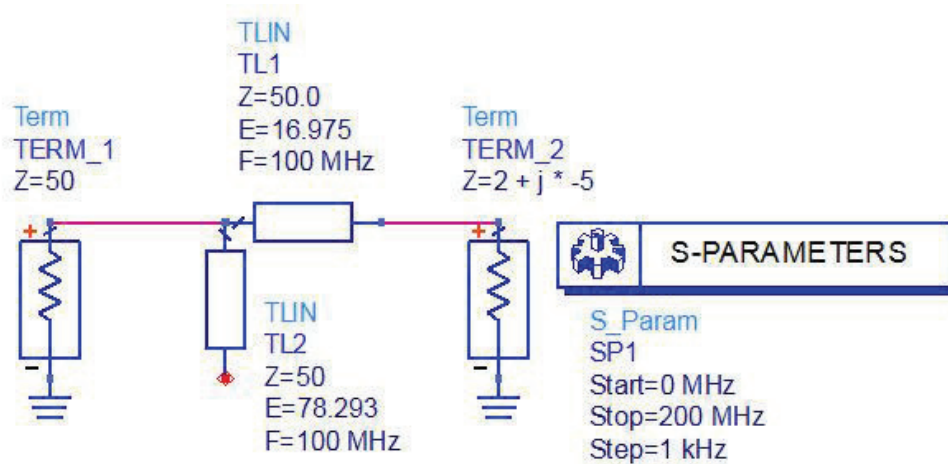
$$B1=(x*t1^2+(r^2+x^2-1)*t1-x)/(RS*(r^2+x^2+t1^2+2*x*t1))$$

$$B2=(x*t2^2+(r^2+x^2-1)*t2-x)/(RS*(r^2+x^2+t2^2+2*x*t2))$$

$$so1=360*(\pi-\text{atan}(RS*B1))/(2*\pi)$$

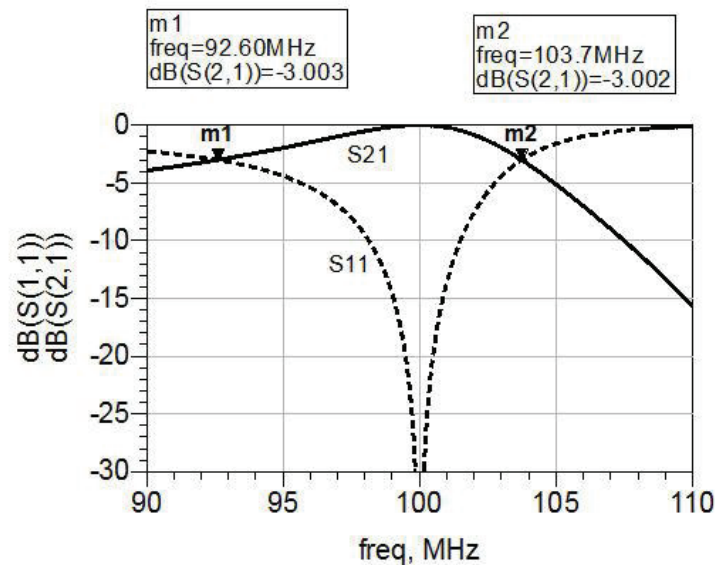
$$so2=-360*\text{atan}(RS*B2)/(2*\pi)$$

The calculation results show that the problem has two solutions. Either solution can be used in the design of the single-stub matching network. However, for both line and stub, usually the shorter line lengths are chosen. For this example we select the shorter electrical lengths in the second solution;  $d2$  for the line and  $so2$  for the open-circuited stub. The schematic of the single-stub matching network is shown in Figure 6-9.



**Figure 6-9** Schematic of the single-stub matching network

Simulate the schematic and display S11 and S21 in a rectangular plot.



**Figure 6-10** Response of the single-stub matching network

Notice that the matching bandwidth at 3 dB return loss is about 11% while at 20 dB is only 1%. This suggests a very narrow matching bandwidth.

## Graphical Design of Single-Stub Matching Networks

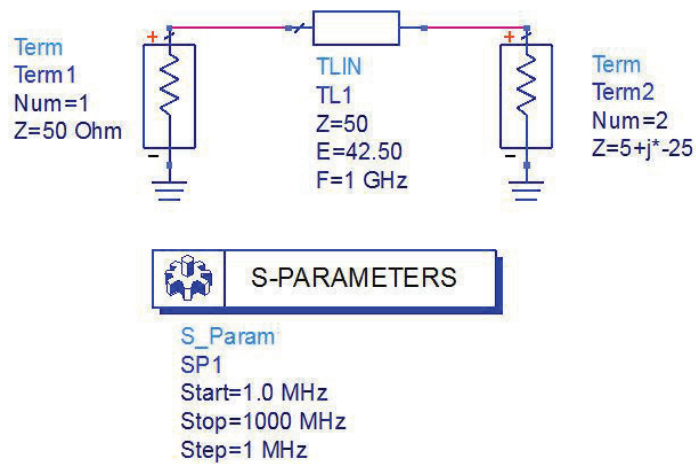
In this section a  $50\ \Omega$  source will be matched to a  $5 - 25j\ \Omega$  load at a frequency of 1000 MHz. Both open-circuited and short-circuited shunt stubs will be considered.

**Example 6 - 6:** Design a single-stub network to match the  $5 - j25\ \Omega$  load impedance to  $50\ \Omega$  source resistor. Use an open-circuited stub.

**Solution:** Create a new workspace in ADS and open a new schematic window. Insert the S\_Params Template and add a series transmission line (TLIN) between the source and load terminations. Wire up the components and set the component values as shown in Figure 6-11.

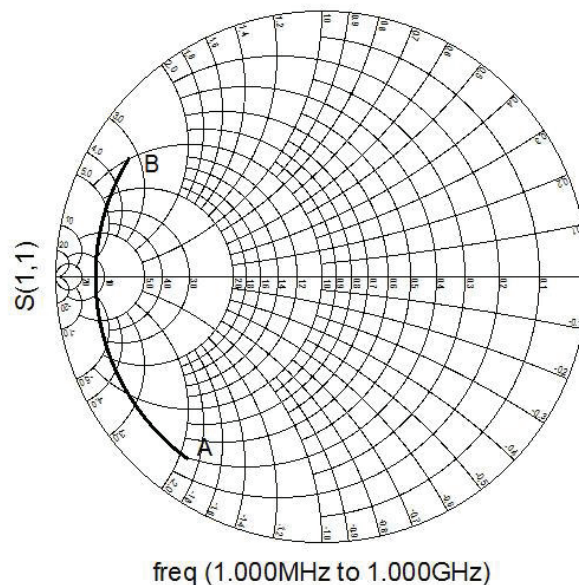
Because we intend to place a shunt element after the series transmission line, enable the admittance circles on the Smith Chart. Make the length of the line tunable and tune it to move the load impedance (point A) to

intersect the unit conductance circle on the top half of the Smith chart (point B). As the schematic shows, an electrical length of  $42.5^\circ$  places the impedance on the unit conductance circle at point B.



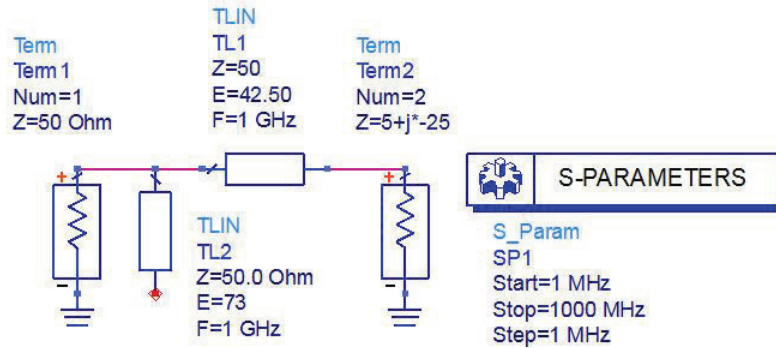
**Figure 6-11** Adding series transmission line (electrical length= $42.5^\circ$ )

The movement of the impedance from point A to point B is displayed in Figure 6-12.



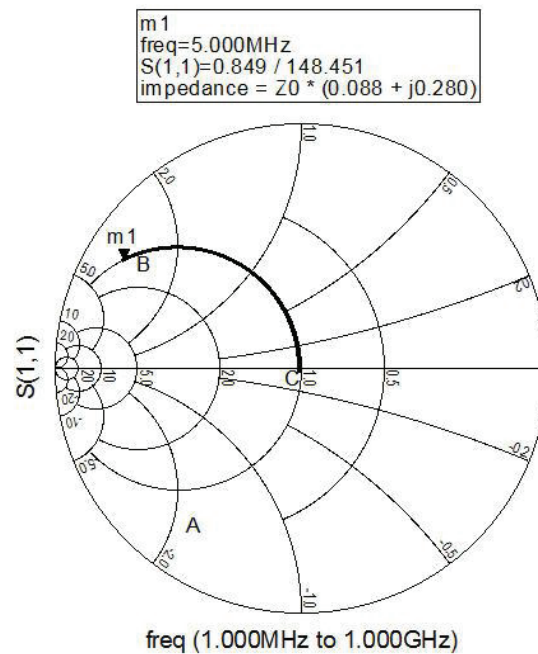
**Figure 6-12** Point B on the unit conductance circle (electrical length= $42.5^\circ$ )

At point B the normalized impedance is  $z = 0.088 + 0.28j \Omega$ . To move point B to the center of Smith chart, add an open-circuited shunt transmission line and tune the length of the line until the impedance moves to the center of the chart ( $50\Omega$ ). As the schematic of Figure 6-13 shows, a  $73^\circ$  length of transmission line would be required.



**Figure 6-13** Adding open-circuited shunt transmission line ( $73^\circ$ ) to network

The movement of point B to the center of the Smith chart is shown in Figure 6-14.



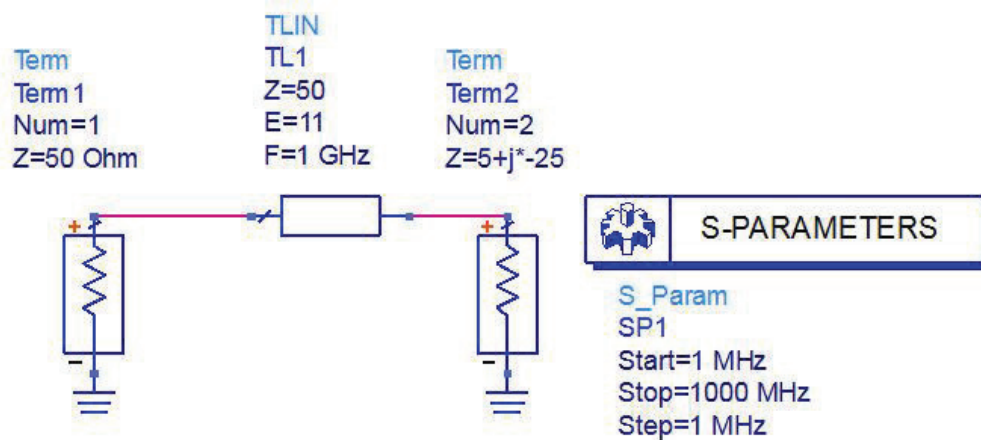
**Figure 6-14** Moving point B to the center of Smith chart ( $73^\circ$ )

## 6.5.2 Graphical Design Using Short-Circuited Stubs

In this Section a short-circuited shunt stub will be used.

**Example 6-7:** Consider matching the  $5 - j25 \Omega$  load impedance to  $50 \Omega$  source resistor using short-circuited shunt stubs.

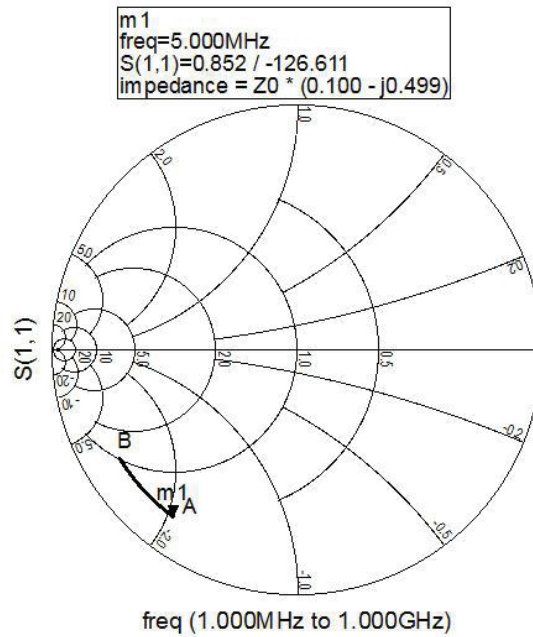
**Solution:** A short-circuited shunt transmission line can be used to perform the function of the shunt stub. Using a short-circuited shunt transmission line, the series transmission line should intersect the unit conductance circle on the bottom half of the Smith Chart. Add a series transmission line of  $11^\circ$  electrical length to move the impedance at point A to intersect the unit conductance circle at point B as shown in Figure 6-16. Then add the short-circuited shunt transmission line as shown in Figure 6-15. Tune the length to  $17^\circ$  to move the impedance to the center of the Smith Chart.



**Figure 6-15** Adding series transmission line ( $11^\circ$ ) to  $5 - j25 \Omega$

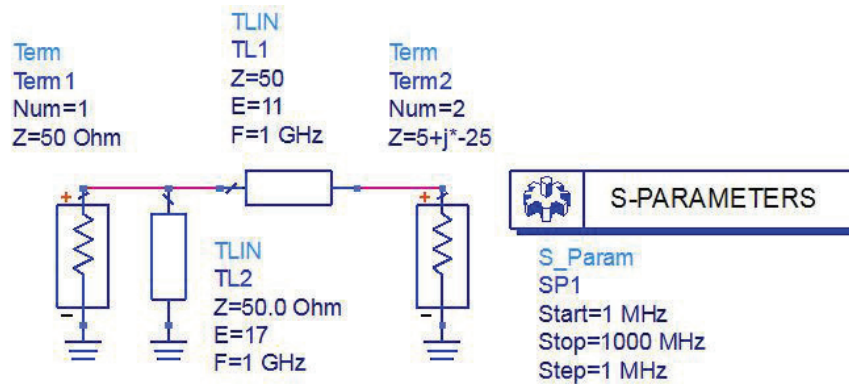
Simulate the schematic and notice the movement of the impedance from point A to point B.





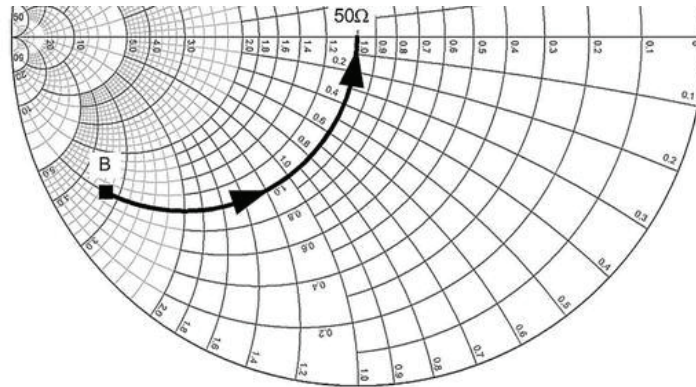
**Figure 6-16** Adding series transmission line ( $11^\circ$ ) to  $5-j25 \Omega$

To move point B to the center of Smith chart, add a short-circuited shunt transmission line and tune the length of the line until the impedance moves to the center of the chart ( $50\Omega$ ). As the schematic of Figure 6-17 shows, a  $17^\circ$  length of transmission line would be required.



**Figure 6-17** Adding short-circuited shunt transmission line ( $17^\circ$ )

Simulate the schematic and notice the movement of the impedance from point B to the center of Smith chart.



**Figure 6-18** Adding short-circuited shunt transmission line ( $17^\circ$ )

## Design of the Cascaded Matching Networks

When the impedance of the load and the source are both complex functions, we can define a virtual resistor and design single-stub networks to match the complex impedances to the virtual resistor. The final matching network is obtained by cascading the single-stub matching networks. The procedure is demonstrated in the following example.

**Example 6-8:** Design single-stub networks to match a complex load  $Z_L = 10 - j5 \, \Omega$  to a complex source  $Z_S = 50 - j15 \, \Omega$  at 100 MHz. Calculate the electrical lengths of the lines and the fractional bandwidths of the matching network at 3 dB and 20 dB return loss.

The procedure follows:

1. Find the intermediate resistor,

$$R_1 = \sqrt{R_L R_S} = \sqrt{(10)(50)} = 22.36 \, \Omega$$

2. Design a single-stub matching network between the intermediate resistor and the load impedance

3. Design a single-stub matching network between the intermediate resistor and the source impedance
4. Cascade the two matching networks

**First Solution:** Design the single-stub matching network between the load impedance and the intermediate resistor

1. Enter design parameters with RS as the intermediate resistor

$$RS=22.36; RL=50; XL=-15; f=100e6, r=RL/RS, x=XL/RS$$

2. Use Equations (6-18) to (6-29) in Appendix C to calculate t1, t2, d1 and d2, B1, B2, so1 and so2.

$$t1=(x+\sqrt{r(r^2+x^2-2r+1)})/(r-1)$$

$$t2=(x-\sqrt{r(r^2+x^2-2r+1)})/(r-1)$$

$$d1=360*(\text{atan}(t1))/(2*\pi)$$

$$d2=360*(\pi+\text{atan}(t2))/(2*\pi)$$

$$B1=(x*t1^2+(r^2+x^2-1)*t1-x)/(RS*(r^2+x^2+t1^2+2*x*t1))$$

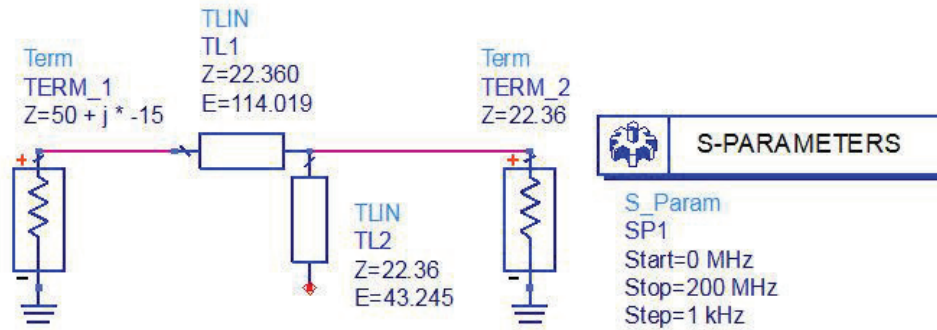
$$B2=(x*t2^2+(r^2+x^2-1)*t2-x)/(RS*(r^2+x^2+t2^2+2*x*t2))$$

$$so1=360*(\pi-\text{atan}(RS*B1))/(2*\pi)$$

$$so2=-360*\text{atan}(RS*B2)/(2*\pi)$$

From calculation results we select d2=114.019 and so2=43.245.

Design of the first matching network is shown in Figure 6-19.



**Figure 6-19** Schematic of the first matching network

**Example 6-9:** Design the single-stub matching network between the source impedance and the intermediate resistor. (Second Solution)

**Solution:** Enter design parameters and normalize the load impedance

$$RS=22.36; RL=10; XL=-5; f=100e6, r=RL/RS' \quad x=XL/RS$$

Use Equations (6-18) to (6-29) in Appendix C to calculate  $t_1$ ,  $t_2$ ,  $d_1$  and  $d_2$ ,  $B_1$ ,  $B_2$ ,  $so_1$  and  $so_2$ .

$$t_1 = (x + \sqrt{r(r^2 + x^2 - 2r + 1)}) / (r - 1)$$

$$t_2 = (x - \sqrt{r(r^2 + x^2 - 2r + 1)}) / (r - 1)$$

$$d_1 = 360 * (\pi + \text{atan}(t_1)) / (2 * \pi)$$

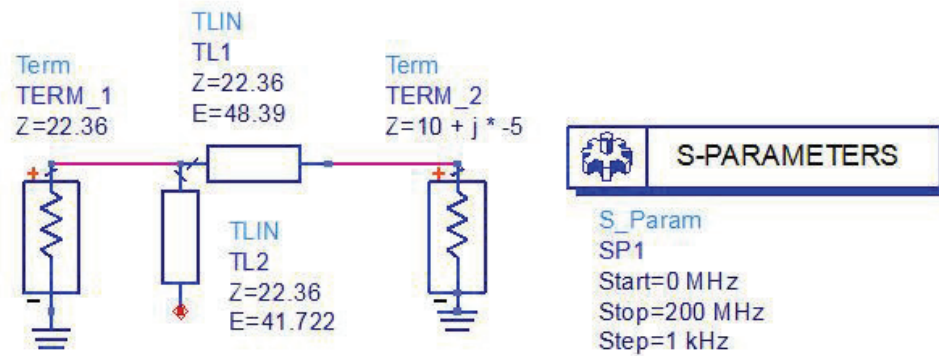
$$d_2 = 360 * (\text{atan}(t_2)) / (2 * \pi)$$

$$B_1 = (x * t_1^2 + (r^2 + x^2 - 1) * t_1 - x) / (RS * (r^2 + x^2 + t_1^2 + 2 * x * t_1))$$

$$B_2 = (x * t_2^2 + (r^2 + x^2 - 1) * t_2 - x) / (RS * (r^2 + x^2 + t_2^2 + 2 * x * t_2))$$

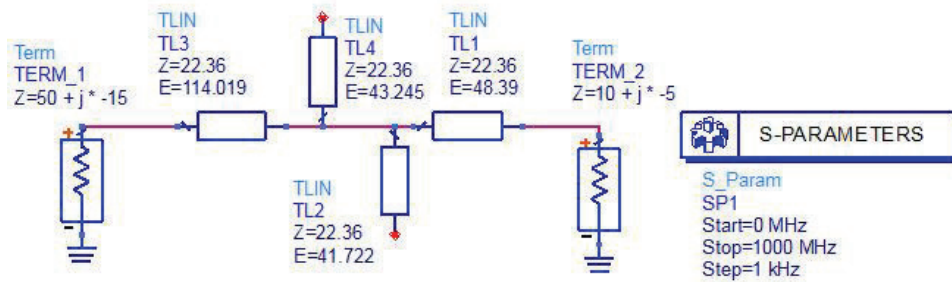
$$so_1 = 360 * (\pi - \text{atan}(RS * B_1)) / (2 * \pi)$$

$$so_2 = -360 * \text{atan}(RS * B_2) / (2 * \pi)$$



**Figure 6-20** Schematic of the second matching network

Now cascade both line and stub matching networks to obtain the complete matching network, as shown in Figure 6-21.



**Figure 6-21** Cascading single-stub matching networks

The simulated response of the complete matching network is shown in Figure 6-22. Notice that the matching network has about 100 % fractional bandwidth at 3 dB return loss and 47% fractional bandwidth at 12 dB return loss. The measured 3 dB bandwidth between the markers is exactly the same as calculated by the equations.

Add markers m1 and m2 at 3 dB return loss to measure the 3 dB matching bandwidth. Notice it agrees well with the simulated BW3DB of 102.7 MHz.

$$121 - 18.27 = 102.73 \text{ MHz}$$

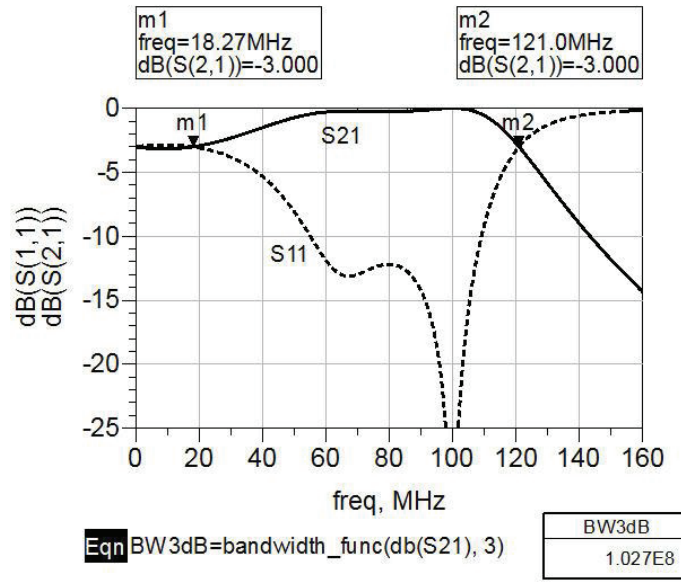


Figure 6-22 Response of the cascaded matching network

## Broadband Design of the Quarter-Wave Matching Network

In Examples 6.2-1 the bandwidth of a single quarter-wave transformer matching network is less than 10 % which is considered to be a narrow-band matching network. We can increase the bandwidth by cascading two or more quarter-wave transformers to achieve a broadband matching network. To analytically design a broadband matching network with  $N$  quarter-wave transformers first use Equation (6-32) to calculate the characteristic impedance of each quarter-wave transformer then cascade all the sections into one matching network. Let  $R_s$  and  $R_L$  be the source and load impedances to be matched by the  $N$  quarter-wave transformation network. The characteristic impedance of each section can be calculated from the following equation.

$$Z_n = R_s(r)^{(2n-1)/2N} \quad n = 1, 2, \dots, N$$

Where  $r = R_L/R_s$  is the normalized load resistance and  $N$  is the number of quarter-wave transformers.

**Example 6-10:** Design a three-section quarter-wave transformer network to match a load resistance  $R_L = 2 \Omega$  to a source  $R_S = 50 \Omega$  at 100 MHz.

- (a) Calculate the characteristic impedance of the quarter-wave transformers, the Q factor and the fractional bandwidths of the matching network at 3 and 20 dB return loss.
- (b) Display the simulated response and compare the measurements with the measurements of single quarter-wave transformer matching network.

**Solution:** (a) Using the above equation for  $Z_n$  the characteristic impedances of the 3-section quarter-wave transformers are:

$$Z_1 = R_S(r)^{1/2N} = 50(0.04)^{1/6} = 29.24$$

$$Z_2 = R_S(r)^{3/2N} = 50(0.04)^{3/6} = 10.00$$

$$Z_3 = R_S(r)^{5/2N} = 50(0.04)^{5/6} = 3.42$$

The intermediate resistors can be obtained from equation below

$$R_n = R_S(r)^{\frac{n}{N}} \quad n = 1, 2, 3, \dots, N-1$$

Therefore, the two intermediate resistors are:

$$R_1 = 50(0.04)^{\frac{1}{3}} = 17.1 \Omega$$

$$R_2 = 50(0.04)^{\frac{2}{3}} = 5.848 \Omega$$

The schematic of the three-section quarter-wave matching network is shown in Figure 6-23.

1. Enter design parameters and normalize the load impedance

$R_S=50$ ;  $R_L=2$ ;  $f=100\text{e}6$ ,  $r=R_L/R_S$ ,  $N=3$

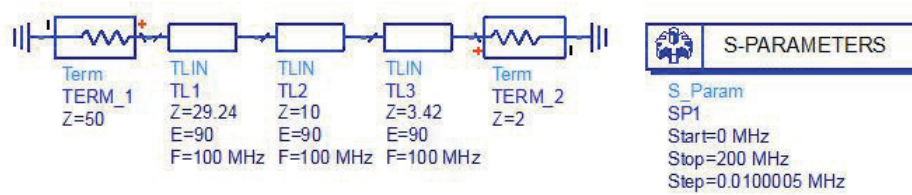
3. Calculate characteristic impedances and intermediate impedances

$$Z_1 = R_S * r^{(1/(2*N))}$$

$$Z_2 = R_S * r^{(3/(2*N))}$$

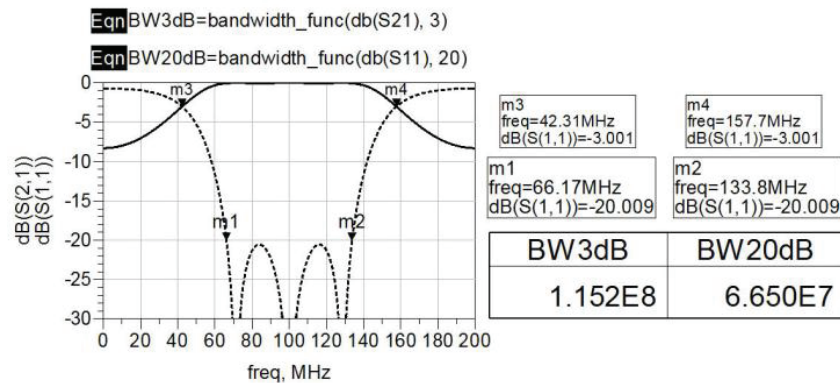
$$Z_3 = R_S * r^{(5/(2*N))}$$

$$R_1 = R_S * r^{(1/N)}; R_2 = R_S * r^{(2/N)}$$



**Figure 6-23** Three-section quarter-wave matching network

The simulated response of the matching network is shown in Figure 6-24.



**Figure 6-24** Response of the cascaded matching network

Figure 6-24 shows that the simulated bandwidth at the 3 dB return loss is:

$$BW_{3dB} = 157.7 - 42.3 = 115.4 \text{ MHz}$$



$$FBW_{3dB} = \frac{115.4 \times 100}{\sqrt{(157.7)(42.3)}} = 140.8 \%$$

Similarly the simulated bandwidth at the 20 dB return loss is:

$$BW_{20dB} = 133.8 - 66.2 = 67.6 \text{ MHz}$$

$$FBW_{20dB} = \frac{67.6 \times 100}{\sqrt{(133.8)(66.2)}} = 71.8 \%$$

The measured Q factor for the matching network is,

$$Q = \frac{1}{1.408} = 0.71$$

The wide bandwidth and lower Q factor is an indication that by adding two more quarter-wave sections the Q factor has reduced to less than one half and the 3-dB bandwidth has more than doubled compared to a single quarter-wave matching network.

**Example 6-11:** Design a broadband quarter-wave network to match a complex load,  $Z_L = 10 - j5 \Omega$  to  $50 \Omega$  source impedance at 100 MHz. Calculate the bandwidth at 20 dB return loss and compare with the bandwidth of a single-stub matching network.

**Solution:** To design a broadband matching network between a resistive source and complex load impedance, first we design a 5 section matching network between the real source and the real part of the load impedance. Then we replace the quarter-wave transformer adjacent to the load with a single-stub network that matches the complex load to the real resistor. The procedure follows:

1. Enter design parameters and normalize the load impedance  
 $RS=50$ ;  $RL=10$ ;  $f=100\text{e}6$ ;  $r=RL/RS$ ;  $N=5$

## 2. Calculate characteristic impedances and intermediate resistors

$$Z1=RS*r^{(1/(2*N))} = 42.567$$

$$Z2=RS*r^{(3/(2*N))} = 30.852$$

$$Z3=RS*r^{(5/(2*N))} = 22.361$$

$$Z4=RS*r^{(7/(2*N))} = 16.207$$

$$Z5=RS*r^{(9/(2*N))} = 11.746$$

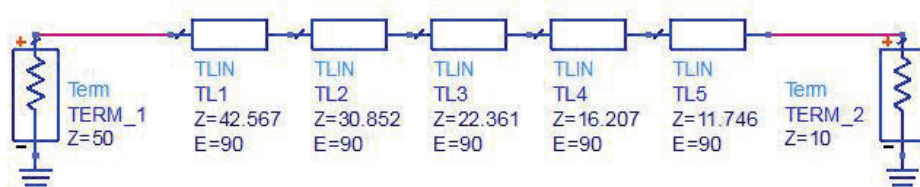
$$R1=RS*r^{(1/N)} = 36.239$$

$$R2=RS*r^{(2/N)} = 26.265$$

$$R3=RS*r^{(3/N)} = 19.037$$

$$R4=RS*r^{(4/N)} = 13.797$$

The calculation results show that the matching network is:



**Figure 6-25** Five-section quarter-wave transformer matching network

Next replace the quarter-wave section adjacent to the load with a single-stub matching network between  $R4 = 13,795$  Ohm and  $Z_L = 10 - j5$  Ohm. Based on following calculations, the single-stub matching network is shown in Figure 6-26.

1. Enter design parameters and normalize the load impedance

$R_S=13.797$ ;  $R_L=10$ ;  $X_L=-5$ ;  $f=100\text{e}6$ ,  $r=R_L/R_S$ ,  $x=X_L/R_S$

2. Use the equations in Appendix C to calculate line and stub values

$$t1=(x+\sqrt{r(r^2+x^2-2r+1)})/(r-1)$$

$$t2=(x-\sqrt{r(r^2+x^2-2r+1)})/(r-1)$$

$$d1=360*(\pi+\text{atan}(t1))/(2*\pi)$$

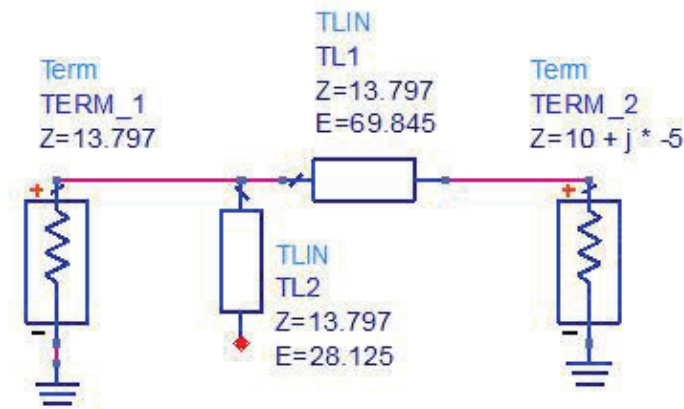
$$d2=360*(\text{atan}(t2))/(2*\pi)$$

$$B1=(x*t1^2+(r^2+x^2-1)*t1-x)/(R_S*(r^2+x^2+t1^2+2*x*t1))$$

$$B2=(x*t2^2+(r^2+x^2-1)*t2-x)/(R_S*(r^2+x^2+t2^2+2*x*t2))$$

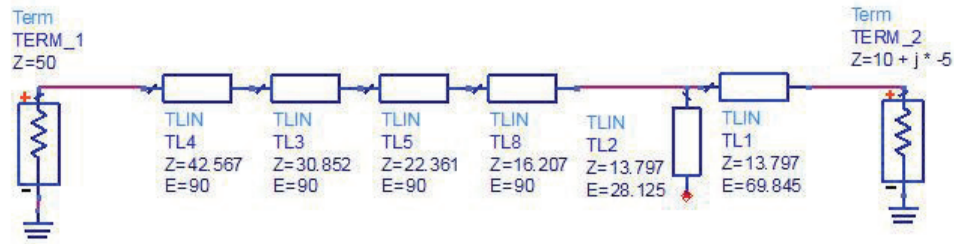
$$so1=360*(\pi-\text{atan}(R_S*B1))/(2*\pi)$$

$$so2=-360*\text{atan}(R_S*B2)/(2*\pi)$$



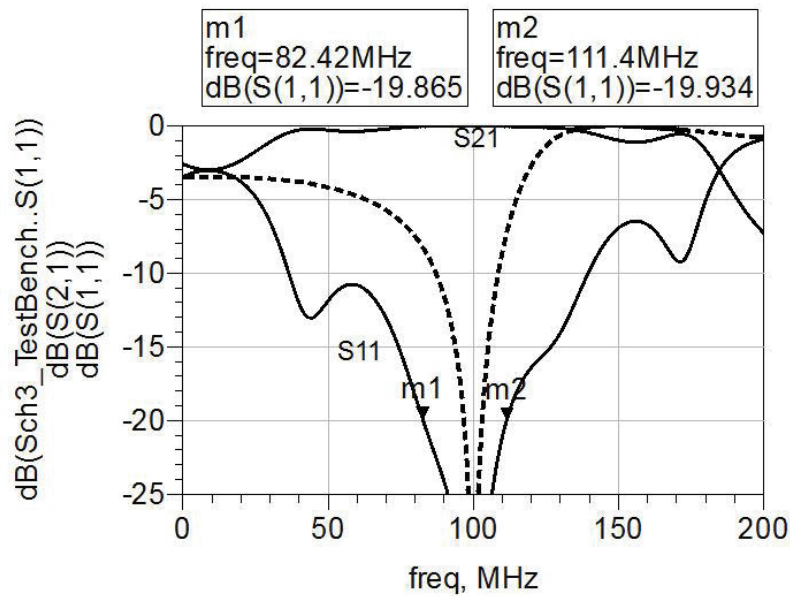
**Figure 6-26** Schematic of the single-stub matching network

Now cascade the line and stub with four quarter-wave networks to form the final design of the matching network as shown in Figures 6-27.



**Figure 6-27** Schematic of the broadband matching network

The simulated response of the matching network is shown in Figure 6-28.



**Figure 6-28** Response of the broadband matching network

Figure 6-28 shows that the bandwidth at 20 dB return loss is:

$$BW_{20\text{dB}} = 111 - 82 = 29 \text{ MHz}$$

And the fractional bandwidth at 20 dB return loss is:

$$FBW_{20\text{dB}} = \frac{\sqrt{29}}{(111).(82)} = 30.3 \%$$

The same numbers for the single-stub matching network are:

$$BW_{20dB} = 102.8 - 96.8 = 6 \text{ MHz}$$

$$FBW_{20dB} = \frac{6 \times 100}{\sqrt{(102.8)(96.8)}} = 6 \%$$

Notice that fractional bandwidth for the broadband at 20 dB return loss is 30.3 % as opposed to only 6 % for the narrowband single-stub matching network. Therefore, we have increased the matching bandwidth at 20 dB return loss by five times over the single-stub matching network.

**Example 6-12:** Design a broadband network to match a complex load,  $Z_L = 150 - j30 \text{ Ohm}$  to  $50 \text{ } \Omega$  source impedance at 100 MHz. Display the simulated response and measure the bandwidth at 20 dB return loss. Compare the results with the single-stub matching network.

**Solution:** Use the same method as in Example 6-10 to design the broadband matching network. The design of the broadband quarter-wave transformer matching network with five quarter-wave sections is shown in Figure 6-29.

1. Enter design parameters and normalize the load impedance

$$R_S=50; R_L=150; f=100\text{e}6, r=R_L/R_S, N=5$$

2. Calculate characteristic impedances and intermediate resistors

$$Z_1 = R_S * r^{(1/(2*N))} = 55.806$$

$$Z_2 = R_S * r^{(3/(2*N))} = 69.915$$

$$Z_3 = R_S * r^{(5/(2*N))} = 86.603$$

$$Z_4 = R_S * r^{(7/(2*N))} = 107.883$$

$$Z5 = RS * r^{(9/(2*N))} = 134.394$$

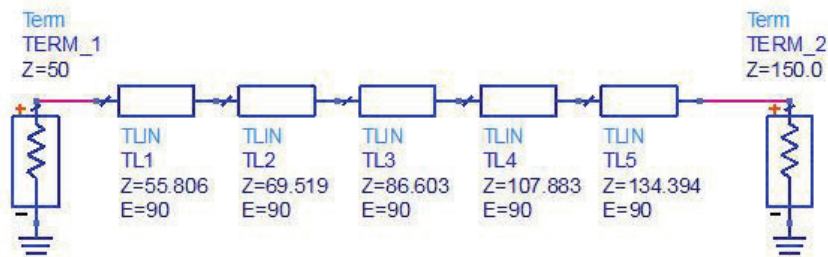
$$R1 = RS * r^{(1/N)} = 62.287$$

$$R2 = RS * r^{(2/N)} = 77.592$$

$$R3 = RS * r^{(3/N)} = 96.659$$

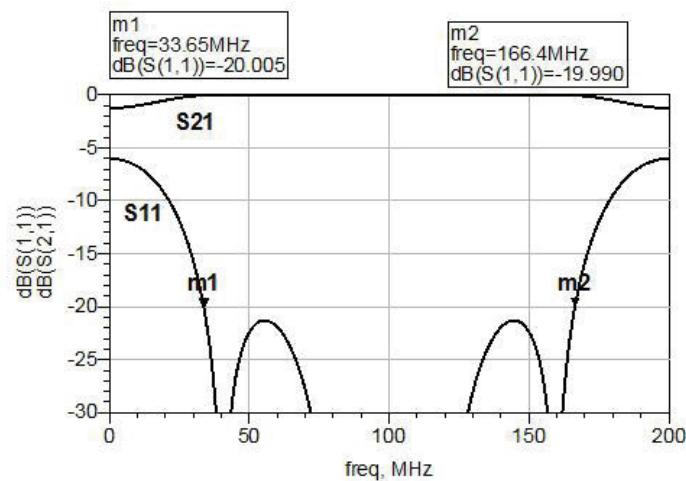
$$R4 = RS * r^{(4/N)} = 120.411$$

The ADS schematic of the five-section quarter-wave matching network is shown in Figure 6-29. The simulated response of the quarter-wave matching network is shown in Figure 6-30. Notice that Z1 is the characteristic impedance for TL1 and so on.



**Figure 6-29** Five-section quarter-wave matching network

The wideband response is shown in Figure 6-30.



**Figure 6-30** Response of the matching network

Next replace the quarter-wave transformer adjacent to the load with a single-stub matching network. The design of the single-stub matching network is shown in the following Equation Editor. Note that the source resistor,  $R_4 = 13.797 \Omega$ , was calculated.

1. Enter design parameters and normalize the load impedance

$$R_S=120.411; R_L=150; X_L=-30; f=100\text{e}6, r=R_L/R_S, x=X_L/R_S$$

2. Use the equations in Appendix C to calculate  $t_1$  and  $t_2$ ,  $d_1$  and  $d_2$ ,

$B_1$  and  $B_2$ ,  $so_1$  and  $so_2$

$$t_1=(x+\sqrt{r(r^2+x^2-2r+1)})/(r-1)$$

$$t_2=(x-\sqrt{r(r^2+x^2-2r+1)})/(r-1)$$

$$d_1=360*(\text{atan}(t_1))/(2*\pi)$$

$$d_2=360*(\pi+\text{atan}(t_2))/(2*\pi)$$

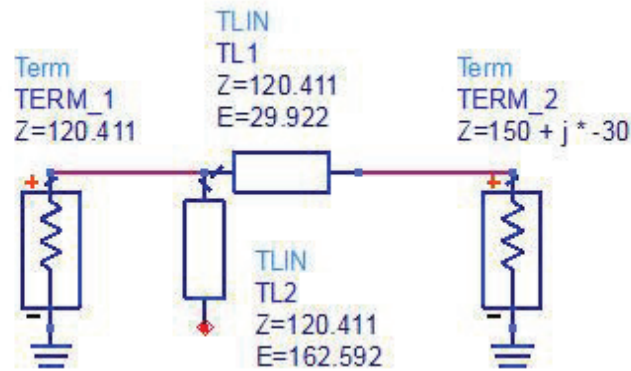
$$B_1=(x*t_1^2+(r^2+x^2-1)*t_1-x)/(R_S*(r^2+x^2+t_1^2+2*x*t_1))$$

$$B_2=(x*t_2^2+(r^2+x^2-1)*t_2-x)/(R_S*(r^2+x^2+t_2^2+2*x*t_2))$$

$$so_1d=360*(\pi-\text{atan}(R_S*B_1))/(2*\pi)$$

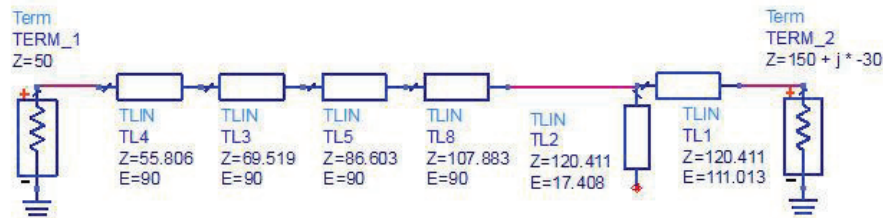
$$so_2d=-360*\text{atan}(R_S*B_2)/(2*\pi)$$

From the solution of the above equations we select  $d_2=111.013$  degrees for the electrical length of the line and  $so_2=17.408$  degrees for the electrical length of stub, as shown in the Figure 6-31.



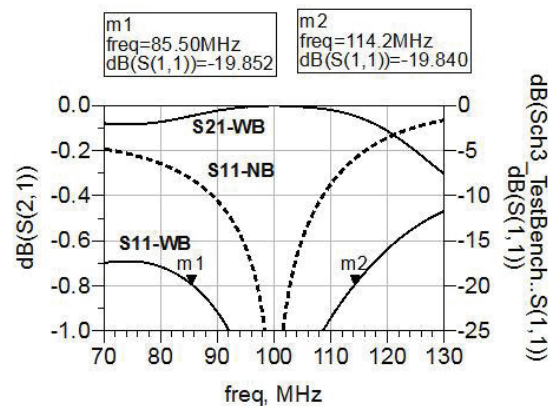
**Figure 6-31** Schematic of the single-stub matching network

Now cascade the two matching networks to form the final design of broadband matching network, as shown in Figure 6-32.



**Figure 6-32** Schematic of the broadband matching network

The simulated response of the quarter-wave matching network is shown in Figure 6-33.



**Figure 6-33** Response of the broadband matching network



Figure 6-33 shows that the bandwidths of the broadband matching network at 20 dB return loss are:

$$BW_{20dB} = 114.2 - 86.0 = 27.8 \text{ MHz}$$

$$FBW_{20dB} = \frac{27.8 \times 100}{\sqrt{(113.8)(86.0)}} = 28.1 \%$$

The same numbers for the narrowband single-stub matching network are:

$$BW_{20dB} = 102.8 - 97.1 = 5.7 \text{ MHz}$$

$$FBW_{20dB} = \frac{5.7 \times 100}{\sqrt{(102.8)(97.1)}} = 5.7 \%$$

Notice that the fractional bandwidth at 20 dB return loss is about 28.1 % as opposed to only 5.7 % for the narrowband matching network. This is an indication that, by adding four quarter-wave sections to the single-stub matching network, we have increased the matching bandwidth at 20 dB return loss nearly five times over a single-stub matching network.

## **References and Further Reading**

- [1] Ali A. Behagi, *RF and Microwave Circuit Design*, Updaed and Revised with 100 Keysight (ADS) Workspaces, Techno Search, Ladera Ranch, California, February 2017
- [2] Keysight Technologies, Manuals for Advanced Design System, *ADS 2016.0 Documentation Set*, EEsof EDA Division, Santa Rosa, California [www.keysight.com](http://www.keysight.com)

- [3] Guillermo Gonzales, *Microwave Transistor Amplifiers – Analysis and Design*, Second Edition, Prentice Hall Inc., Upper Saddle River, NJ.
- [4] Randy Rhea, *The Yin-Yang of Matching: Part 1 – Basic Matching Concepts*, High Frequency Electronics, March 2006
- [5] Steve C. Cripps, *RF Power Amplifiers for Wireless Communications*, Artech House Publishers, Norwood, MA. 1999.
- [6] David M. Pozar, *Microwave Engineering*, Third Edition, John Wiley & Sons, New York, 2005
- [7] R. Ludwig, P. Bretchko, *RF Circuit Design, Theory and Applications*, Prentice Hall, Upper Saddle River, NJ, 2000

## **Problems**

- 6-1. Design a quarter-wave transmission line to match a load resistance  $R_L = 5 \, \Omega$  to a resistive source  $R_S = 75 \, \Omega$  at 500 MHz. Calculate the characteristic impedance of the quarter-wave line, the Q factor and the fractional bandwidths of the matching network at 3 and 20 dB return loss. Display the simulated response and compare the calculations with measurements. For the quarter-wave matching network, calculate the fractional bandwidth and power loss from  $\Gamma = 0.1$  to  $\Gamma = 0.707$ .
- 6-2. Design a quarter-wave transmission line to match a load resistance  $R_L = 100 \, \Omega$  to a resistive source  $R_S = 25 \, \Omega$  at 600 MHz. Calculate the characteristic impedance of the quarter-wave line, the Q factor and the fractional bandwidths of the matching network at 3 dB and 20 dB return loss. Display the simulated response and compare the calculations with measurements. For the quarter-wave matching network, calculate the fractional bandwidth and power loss from  $\Gamma = 0.1$  to  $\Gamma = 0.707$ .

- 6-3. Design a single-stub network to match a load resistance  $Z_L = 5 - j5 \Omega$  to a resistive source  $R_S = 75 \Omega$  at 700 MHz. Calculate the electrical lengths of the matching line and stub and the fractional bandwidths of the matching network at 3 and 20 dB return loss. Display the simulated response and compare the calculations with measurements.
- 6-4. Design a single-stub network to match a load resistance  $Z_L = 100 + j20 \Omega$  to a resistive source  $R_S = 40 \Omega$  at 800 MHz. Calculate the electrical lengths of the matching line and stub and the fractional bandwidths of the matching network at 3 dB and 20 dB return loss. Display the simulated response and compare the calculations with measurements.
- 6-5. Design a single-stub network to match a complex load  $Z_L = 20 + j5 \Omega$  to a complex source  $Z_S = 50 + j20 \Omega$  at 1000 MHz. Calculate the electrical lengths of the matching line and stub and the fractional bandwidths of the matching network at 3 dB and 20 dB return loss. Display the simulated response and verify the calculations with measurements.
- 6-6. Design a three-section quarter-wave network to match a load resistance  $R_L = 5 \Omega$  to a resistive source  $R_S = 50 \Omega$  at 100 MHz. Calculate the characteristic impedance of the quarter-wave line, the Q factor and the fractional bandwidths of the matching network at 3 and 20 dB return loss. Display the simulated response and compare the measurements with the single quarter-wave matching network.
- 6-7. Design a three-section quarter-wave network to match a load resistance  $R_L = 100 \Omega$  to a resistive source  $R_S = 25 \Omega$  at 900 MHz. Calculate the fractional bandwidths of the matching network at 3 and 20 dB return loss. Display the simulated response and compare the measurements with the single quarter-wave matching network.
- 6-8. Design a broadband network to match a complex load,  $Z_L = 25 + j10 \Omega$  to  $75 \Omega$  source impedance at 300 MHz. Measure the

bandwidth at 20 dB return loss and compare it with the results of single-stub matching network.

- 6-9. Design a broadband network to match a complex load,  $Z_L = 100 + j20$  to  $40 \Omega$  source impedance at 1200 MHz. Display the simulated response and measure the bandwidth at 20 dB return loss. Compare the results with the single-stub matching network.